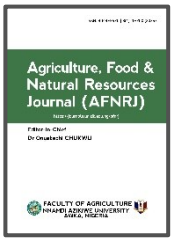
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Original Article

Comparison of estimation methods for diameter distribution models of *Tectona grandis* Linn.f. in Omo Forest Reserve, Nigeria

OPEN ACCESS

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ABSTRACT

Accurate tree diameter characterisation is fundamental to growth and yield modelling and critical for sound decision-making in sustainable forest management. Despite its importance, there is a paucity of comparative studies on optimal probability density functions and parameter estimation approaches for Tectona grandis stands within the Omo Forest Reserves, Nigeria. This study evaluated the comparative performance of Maximum Likelihood Estimation (MLE) and Methods of Moments Estimation (MME) for diameter characterization using Diameter at breast height ≥ 10cm. Data from 658 tree samples aged from 9 to 15 years were used to fit six diameter distribution models, evaluated by log-likelihood (loglik), Akaike information criterion (AIC), Bayesian information criterion (BIC) and Kolmogorov-Smirnov (KS) statistics and graphical appraisal of the resultant probability density function. Results showed that the 3-parameter Weibull distribution consistently provided the best fit across both methods, with Loglik, AIC, BIC, and KS values of -1866.03, 3738.07, 3751.54, and 0.0399, respectively, under MLE. Similarly, under MME, it achieved AIC, BIC and KS statistics of 3747.52, 3760.98 and 0.0506, respectively outperforming all competing models. Johnson's SB ranked second under both methods, followed by generalized extreme value and Gumbel distributions, while the logistics and 2-parameter Weibull performed poorly. The stand structure for the pooled age revealed an inverse J-shaped mimicking uneven-aged structure signifying intense competition, with many trees in the smallest diameter classes. Both estimation methods are statistically viable, with MLE demonstrating superior flexibility and robustness. Consequently, the 3-parameter Weibull distribution estimated by MLE is recommended for Tectona grandis diameter characterisation in the study area.

KEYWORDS: *Maximum likelihood, Moments of moments, Probability density function, Weibull parameters*

INTRODUCTION

Individual tree growth potential is often estimated via diameter distribution models, which are fundamental to forest management decision-making, optimum harvesting regimes, prescribed silvicultural treatments and planning. Accurately characterising tree diameter is essential in predicting the stand's structure, volume and yield potential (Egonmwan & Ogana, 2020; Popoola *et al.,* 2026) as well as forecasting the future growth of forest stands. In Nigeria, *Tectona grandis* plantation establishment has become prominent because of the wood species quality, fast growth rate, importance and the ongoing increase in its demand (Popoola *et al.,* 2025).

Diameter distribution models enable forest managers to estimate timber quantity available by size class. It also an indicator for forest suitability, sustainability and projecting the density of smaller trees as a replacement status for the population of the current larger trees in a stand. Since the pioneering work of Bailey and Dell (1973), diameter distribution concepts using probability density functions have become standard tools for describing frequency of trees across diameter classes in even-aged stands. Several probability

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density functions including the Weibull, Johnson's SB, Gamma, gumbel, generalized extreme value, beta and log-normal distributions have been widely employed (Bailey & Dell, 1973; Hafley & Schreuder, 1977; Gorgoso-Varela and Rojo-Alboreca, 2014; Ogana *et al.,* 2016; Gorgoso-Varela *et al.,* 2024; Popoola *et al.,* 2025) to describe diameter structure in even-aged stands.

The predictive performance of any diameter distribution models depends not only on the suitability of the selected probability distribution but also on the parameter estimation techniques been used. According to Cao (2004), accurate characterisation of the stand structure and tree frequencies across diameter classes is directly influenced by the selected model parameter estimation methods. Common estimation techniques include the method of moments (MME), maximum likelihood estimation (MLE), percentile methods, Bayesian estimation and least-squares approaches. Comparative evaluation of each method offers distinct advantages and drawbacks depending on sample size, dataset characteristics and the distribution being modelled, suggesting that no single estimation method consistently outperforms others across all forest conditions (Gorgoso-Varela *et al.,* 2024).

Maximum likelihood estimation is one of the most commonly applied parameter estimation techniques in forestry because of its desirable asymptotic properties (Diamantopoulou *et al.,* 2015). Under standard assumptions, MLE provides efficient parameter estimates with minimum variance as the sample size increases (Diamantopoulou *et al.,* 2015; Schütz & Rosset, 2020). However, its performance depends on appropriate initial values and numerical optimisation, which may fail to converge for small sample sizes. In contrast, the method of moments (MME) is computationally simpler, estimating parameters directly from sample moments such as the mean and variance, making it suitable for routine forest inventory applications (Gorgoso-Varela *et al.,* 2024) and large-scale forest assessments. Previous studies have shown that the relative performance of estimation methods used varies by species, stand structure, stand age, density and sample size (Borders & Patterson, 1990; Gorgoso-Varela *et al.,* 2024). Despite extensive research on diameter distribution modelling across various species and forest types, several authors have argued in favour of different probability density functions as the best in fitting diameter of trees, however, no single methods consistently outperform others across all forest conditions (Aigbe & Omokhua, 2014; Ogana *et al.,* 2016; Bassey & Adekunle, 2022; Gorgoso-Varela *et al.,* 2024). For example, Ogana *et al.,* 2016 identified the Johnson's SB as the best models for *Gmelina arborea* plantations in Omo Forest reserve, whereas Aigbe & Omokhua (2014) reported the 3-paramter Weibull distribution as the most suitable for trees diameter distribution in Oban Forest reserve. Similarly. Bassey & Adekunle (2022) found Dagum four-parameters distribution to best characterise the diameter distributions in Ukpon Rainforest Reserve, Cross-river State, Nigeria. These inconsistences highlight the need for species- and site-specific evaluation of diameter distribution models and their parameters estimation methods.

In Nigeria, plantation establishment of *Tectona grandis* has become increasingly prominent due to the species superior wood quality, fast growth rate, economic importance and rising domestic and international market demand (Popoola *et al.,* 2025). Consequently, sustainable management of the *Tectona grandis* plantation requires accurate information on the stand structure, volume and yield potential, which can be obtained through reliable diameter distribution models. The dearth of up-to-date information on the comparative evaluation of different probability density functions and parameters estimation approaches specific for *Tectona grandis* necessitated this study. Therefore, this study was conducted to compare the Methods of Moments and Maximum Likelihood Estimation for fitting Gumbel, Johnson's SB, Logistics, Generalized Extreme Value, 3-Paramter Weibull and 2-Parameter Weibull distributions to characterise the diameter distribution of *Tectona grandis* stands in Area J4, Omo Forest Reserve, Nigeria.

MATERIALS AND METHOD

Study Area

This study was conducted in Omo Forest Reserve (OFR), located in Ijebu East and North Local Government Areas of Ogun State, Southwestern Nigeria. The reserve covers approximately 130,500 hectares and lies between Latitudes 6º 35' to 7º 05' N and Longitudes 4º 10' to 4º 35' E (Figure 1). Geographically, the reserve is situated about 120 km east of Abeokuta, about 135 km northeast of Lagos and about 80 km east of Ijebu-Ode (Salako *et al.,* 2020). The area is characterised by a humid tropical climate, with a mean annual rainfall ranging from 1,600 to 2,000 mm, with two annual peaks in June and September. The average monthly temperature ranges from 23°C in July to 32°C in February. The soils are predominantly loamy to sandy, with gravelly layers commonly occurring at depths of 30-60 cm.

Data Collection, Processing, and Analysis

Field-based mensuration approach was used to establish fifteen (15) Temporary Sample Plots (TSP) of 0.04 ha across three different age series stratum (9, 13, 15 years old) using simple random sample techniques. Within each sampled plots, dendrometric variables of individual trees were measured. The Diameter at Breast Height (Dbh) at 1.3 m of trees ≥ 10 cm were measured using a diameter tape following the standard rules for Dbh measurements. For trees with abnormality at breast height such as swelling, knots and deformation, measurements were made above the abnormalities while also considering bifurcation of trees during measurement. All statistical analyses, model development and fitting, and graphical presentations were performed using R statistical software (version 4.3.2). While using Microsoft Excel for that entering, clearing and some computation.

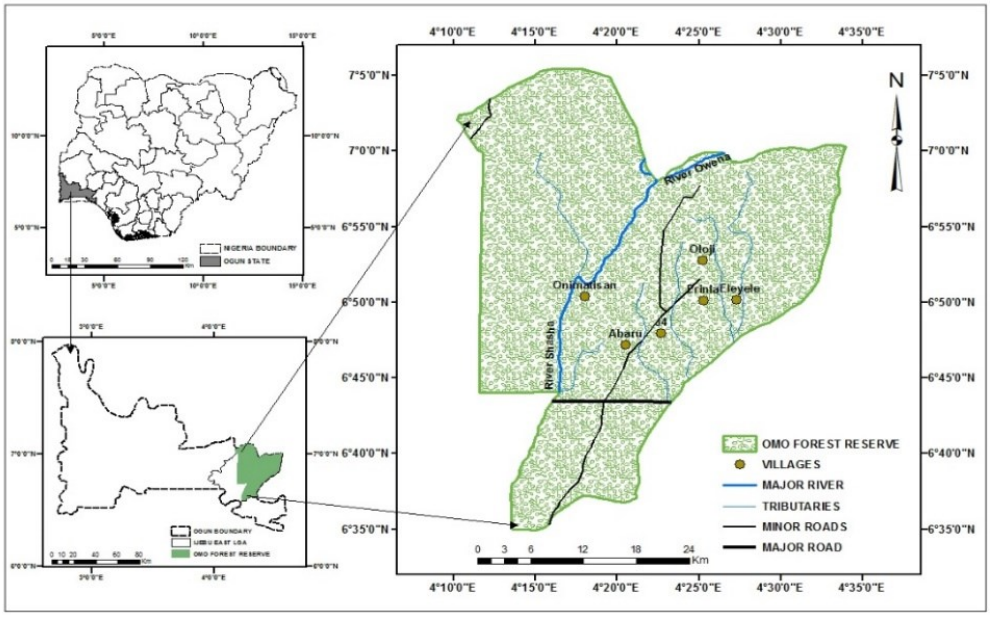


Figure 1: Map of Omo Forest Reserve. Source: Adesoye & Ezenwenyi (2014)

Diameter Distribution Models and Fitting

Six (6) candidate models having 2 to 4-parameters estimates were considered as candidate models for the diameter characterisation in order to avoid over parameterisation. Priority was given to models that has been established to be success in characterising diameter distribution of *Tectona grandis* plantation (Binoti *et. al.,* 2012; Egonmwan and Ogana, 2020; Gorgoso-Varela *et al.,* 2024) and that does not violate biological reality. The models considered were: Gumbel, 4-paramter Johnson's SB (Johnson, 1949), Logistics, Generalized Extreme Value (Nakajima *et al.,* 2012), 3-Paramter Weibull and 2-Parameter Weibull (Weibull, 1951). The models forms and their Probability Density Functions (PDF) are mathematically represented in Table 1. The Parameters of the models were estimated via maximum likelihood estimation and methods of moments, ensuring reproducibility and numerical stability.

Akaike's Information Criterion (AIC) (Akaike, 1974), Schwarz's Bayesian Information Criterion (BIC) (Schwarz, 1978), Log-likelihood (loglik.) and Kolmogorov-Smirnov statistics (KS) were the four goodness-of-fit statistics computed for each method of evaluation. They are mathematically defined as follows:

$$KS_i = max\{max_{1\le i\le n_i}[(j/n_i) - u_j], max_{1\le i\le n_i}[u_j - (j-1/n_i)]\} \quad (1)$$

$$loglik. = nln\left(\frac{R}{n}\right) + n + nln(2\pi) \quad (2)$$

$$BIC = n\ln\left(\sum_{i=1}^{i=n}\left(\frac{Y_{ijk}-\hat{Y}_{ijk}}{n}\right)^2\right) + p \times ln(n) \quad (3)$$

$$AIC = -n\ln\left(\sum_{i=1}^{i=n}\left(Y_{ijk} - \hat{Y}_{ijk}\right)^2\right) + 2p \quad (4)$$

Where: $\overline{Y}_i$ = Arithmetic mean; $\hat{Y}_{ijk}$ = Predicted values for the ith observations; Y_{ijk} = Actual observation for the ith observations on sample plot; n_i = Numbers of observation; and p = Numbers of estimated parameters;

$$u_j = F(x_j) = 1 - e^{\left(-\frac{(x-a)}{b}\right)^c} \quad (5)$$

The KS statistics was used to calculate the maximum absolute difference between the observed cumulative distribution function and the predicted cumulative distribution function (Jung *et al.,* 2025). The models with the highest log-likelihood values and lowest AIC, BIC and KS values was selected as the best-fit models. Relative ranking for the above selection criteria was used in assigning weighted score to the models. Then, the Goodness-of-fit statistics were average for the four to avoid over-reliance on any single criterion.

Table 1: Probability Density Function of Models Examined and Estimation Methods

S/N	Model Functions	Probability Density Function	Parameter Estimation Method	
			Methods of Moments	Maximum Likelihood
1	3-parameter Weibull	$f(x) = \frac{\alpha}{\beta}\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} e^{\left(-\frac{(x-\gamma)}{\beta}\right)^{\alpha}}$ $x \geq \alpha, \beta > 0, \gamma > 0$ β = Scale; α = Shape; γ = Location	$g_i = \frac{\Gamma(1+3/\hat{\alpha})-3\Gamma_1\Gamma_2+2\Gamma_1^3}{(\Gamma_2-\Gamma_1^3)^{\frac{3}{2}}}$; $\hat{\beta} = \frac{s}{\sqrt{\Gamma_2-\Gamma_1^2}}; \hat{\gamma} = \bar{d} - \hat{\beta}\Gamma_1$ σ =Variance; Γ(·)=gamma function; $\bar{d}$ = Diameter arithmetic mean	$\ell = -nln\alpha - nln\beta + (\alpha - 1)\sum ln\left(\frac{d_i-\gamma}{\beta}\right) - \sum\left(\frac{d_i-\gamma}{\beta}\right)^{\alpha}$
2	2-Parameter Weibull	$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{\left(-\frac{(x)}{\beta}\right)^{\alpha}}$ $x \geq \alpha, \beta > 0,$ β = Scale; α = Shape;	$\frac{s}{\bar{d}} = \sqrt{\frac{\Gamma(1+2/\hat{\alpha})}{\Gamma^2(1+1/\hat{\alpha})}} - 1;$ $\hat{\beta} = \frac{\bar{d}}{\Gamma(1+1/\hat{\alpha})};$ σ =Variance; Γ(·)=gamma function; $\bar{d}$ = Diameter arithmetic mean	$\ell = -nln\alpha - n\alpha\ln(\beta)$ $+(\alpha-1)\sum lnd_i - \sum\left(\frac{d_i}{\beta}\right)^{\alpha}$
3	Gumbel	$f(x) = \frac{1}{\beta}e^{\left(-\frac{(x-\mu)}{\beta} - \left(e^{-\frac{(x-\mu)}{\beta}}\right)\right)}$ μ =location; β = Scale	$\hat{\mu} = \bar{d} - \gamma\hat{\beta}; \hat{\beta} = \frac{\sqrt{6}s}{\pi}$	$\ell = -nln\beta - \sum\frac{d_i-\mu}{\beta} - \sum e^{\left(-\frac{d_i-\mu}{\beta}\right)}$
4	Logistics	$f(x) = \frac{e^{-\frac{x-\mu}{s}}}{s\left(e^{-\frac{x-\mu}{s}}\right)^2}$ μ = location; s = scale	$\hat{\mu} = \bar{d}; \hat{s} = \frac{\sqrt{3}s}{\pi}$	$\ell = nln(s) - \sum\frac{d_i-\mu}{s} - 2\sum\ln(1+e^{-\frac{d_i-\mu}{s}})$
5	Johnson's SB	$f(x) = \frac{\delta}{\sqrt{2\pi}}\left(\frac{\lambda}{(\xi+\lambda-x)(x-\xi)}\right)e^{-\frac{1}{2}[\gamma+\delta}$ $\xi < x < \xi + \lambda, ; -\infty < \xi < \infty; -\infty < \gamma < \infty;$ $\lambda > 0 \; and \; \delta > 0$ ξ = location; λ = scale; γ = asymmetry and δ = kurtosis	$\delta \cong \frac{\mu(1-\mu)}{Sd(x)} + \frac{Sd(x)}{4}\left[\frac{1}{\mu(1-\mu)} - 8\right];$ $\gamma \cong \delta ln\left(\frac{1-\mu}{\mu}\right) + \left(\frac{0.5-\mu}{\delta}\right); \mu = \frac{\bar{d}-\xi}{\lambda}; Sd(x) = \frac{\sigma_x}{\lambda}$ $\bar{d}$ = Diameter arithmetic mean; $Sd(x)$= modified Standard (δ) deviation; σ_x= plot diameter standard deviation	$\ell = nln(\delta) - nln(\lambda) - \frac{n}{2}\ln(2\pi) - \sum\ln(z_i) - \sum\ln(1-z_i) - \frac{1}{2}\sum\left(\gamma + \delta ln\left(\frac{z_i}{1-z_i}\right)\right)^2$
6	Generalized Extreme Value	$f(x) = \frac{1}{\sigma}\left(1 + \xi\frac{x-\mu}{\sigma}\right)^{\frac{-1}{\xi-1}} e^{\left(-\left[1+\xi\frac{(x-\mu)}{\sigma}\right]^{-1/\xi}\right)}$ μ = location; σ = scale; ξ = Shape	$\hat{\sigma} = \frac{s-\hat{\xi}}{\sqrt{\Gamma(1-2\hat{\xi})-\Gamma^2(1-\hat{\xi})}}; \hat{\mu} = \bar{d} - \frac{\hat{\sigma}}{\hat{\xi}}\left[\Gamma(1-\hat{\xi}) - 1\right]$	$\ell = nln(\sigma) - \left(\frac{1}{\xi+1}\right)\sum\ln\left(1 + \xi\frac{d_i-\mu}{\sigma}\right) - \sum\left(1+\xi\frac{d_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}$

RESULT

The summary descriptive statistics for all 658 sampled tree stands, for characterising the stand structure, are presented in Table 2. The diameter at breast height (Dbh) ranged from 10.00 cm to 35.20 cm, with a mean and standard deviation (SD) value of 12.6465 cm ± 2.0782, 16.7435 cm ± 4.9718 and 19.4714 cm ± 5.0278 for 6 -, 13- and 15-years old stands, respectively.

The results for the six difference models considered for characterising the diameter distribution of the *Tectona grandis* stands are presented in Table 3 with their parameter estimates and statistics. The shape parameter (α) for the 3-parameter Weibull using the Method of Moments Estimate (MME =1.269) exceeded the Maximum Likelihood Estimate (MLE =1.127) by an approximate value of 12.6%, the largest among the shape parameters followed by 2-parameter Weibull. For the GEV models, the shape (ξ = -0.199), location (μ = 13.640) and scale (σ = 3.403) parameters were the same for the two approaches. In addition, the Gumbel location parameters are similar with a difference of 0.001. The logistics location (μ) parameter for MME (16.317) was consistently higher than MLE (15.768) with both shifting towards larger diameters (Table 3). Also, the 2-parameter Weibull shows a negligible difference of 0.043 cm between the MLE and MME approaches. The two methods strongly describe the same stand structure with a minimum diameter near 10 cm.

The goodness of fit statistics estimated using the MLE and MME approaches are presented in Table 4. For MLE, the 3-parameter Weibull distribution recorded the highest Log-likelihood (Loglik), and lowest Akaike Information Criterion (AIC) and Bayesian Information Criteria (BIC) and smaller Kolmogorov-Smirnov (KS) values of -1866.03, 3738.07, 3751.54 and 0.0399, respectively. The Johnson's SB distribution was ranked second with loglik, AIC, BIC and KS values of -1871.34, 3750.68, 3768.64 and 0.0459, respectively. Next to Johnson's SB is Generalised Extreme value (GEV) distribution with AIC of 3845.73 and KS value of 0.0503 (Table 4).

Similar trends were also observed for the MME with the 3-parameter Weibull having the highest Loglik, lowest AIC and BIC and KS values of -1870.76, 3747.52, 3760.98 and 0.0506, respectively. Next was Johnson's SB model with Loglik, AIC, BIC and KS values of -1877.97, 3763.93, 3781.89, and 0.0459, respectively (Table 4).

For both estimation methods, following Johnson's SB is Generalized Extreme Value (GEV) and Gumbel distributions, whereas the Logistic and 2-parameter Weibull distributions exhibited comparatively poorer performance.

Comparing the two estimation methods, MLE consistently produced better model fits as evident in the higher Loglik, lower AIC and BIC values and smaller KS statistics than MME for most of the models, indicating superior parameter estimation efficiency. The probability density function diameter characterisation graph comparison for each individual models examined are presented in Figure 1(a-b), respectively. The diameter structure distribution pattern for the pooled aged as revealed by the histogram is an inverse J-shaped mimicking uneven-aged structure with stem numbers decreasing with an increase in the diameter classes. The histogram reveals that the youngest stand contributed most of its stems to the small diameter classes while the older ones contributed only a few larger trees. The few trees in the large-diameter classes revealed the extant of competition, site heterogeneity, self-thinning and natural regeneration currently on-going within the stands.

The mean values for the parameter estimate for the two estimation methods (MME and MLE) were also more similar to one another (Table 3), and supported with probability density graph for each model examined as shown in Figure 2(a-f) with strong size inequality distribution leading to the right-skewed of the histograms. Therefore, MLE appears to be more efficient estimation technique for fitting diameter distribution models for the *Tectona grandis* stands in the study area.

Table 2: Diameter at breast Height Summary Statistics for the *Tectona grandis* Stands

Age	N	Mean	Minimum	Maximum	Std. Error	Std. Dev.	Variance
6	215	12.6465	10.0000	20.7000	0.1417	2.0782	4.3189
13	223	16.7435	10.0000	34.5000	0.3329	4.9718	24.7190
15	220	19.4714	10.1000	35.2000	0.3390	5.0278	25.2786

Where: N = Total numbers of measured trees; Std. Dev = Standard Deviation; and Std. Error = Standard Error.

Table 3: Parameter Estimate and Statistics for the Diameter Distribution Models for the *Tectona grandis* Stands

Models	Parameters	MLE Estimate Approach				MME Estimate Approach			
		Value	Min.	Max.	Std. Error	Value	Min.	Max.	Std. Error
3-P Weibull	shape (α)	1.127	0.927	1.267	0.052	1.269	1.165	1.373	0.037
	scale (β)	6.592	5.586	7.389	0.268	6.912	6.222	7.684	0.244
	location(γ)	9.982	9.935	10.000	0.012	9.900	9.900	9.900	0.000
2-P Weibull	shape(α)	3.311	3.057	3.568	0.089	3.553	3.235	3.829	0.100
	scale (β)	18.163	17.559	18.940	0.238	18.120	17.517	18.894	0.237
GEV	shape (ξ)	-0.199	-0.339	-0.073	0.044	-0.199	-0.339	-0.073	0.044
	location(μ)	13.640	13.049	14.149	0.210	13.640	13.049	14.149	0.210
	scale (σ)	3.403	2.992	3.806	0.144	3.403	2.992	3.806	0.144
Logistics	location (μ)	15.768	15.141	16.459	0.217	16.317	15.781	16.985	0.205
	scale (s)	2.869	2.606	3.171	0.093	2.808	2.599	3.118	0.088
Johnson's SB	asymmetry (γ)	1.139	0.794	1.421	0.071	1.073	0.748	1.247	0.056
	kurtosis (δ)	0.766	0.612	0.932	0.044	0.803	0.723	0.868	0.023
	location (ξ)	9.737	9.395	9.910	0.075	9.500	9.500	9.500	0.000
	scale (λ)	26.994	21.165	31.509	0.934	26.200	21.000	26.200	0.464
Gumbel	location (μ)	14.023	13.581	14.496	0.158	14.024	13.513	14.530	0.170
	scale (β)	3.754	3.407	4.116	0.119	3.972	3.675	4.409	0.124

N = 658. Where: Min. = Minimum; Max. = Maximum; Std. Err. = Standard Error of estimate; MLE = Maximum Likelihood Estimate; GEV = Generalized Extreme Value and MME =Methods of Moments Estimate

Table 4: Fit Statistics Model Selection Criteria for the Diameter Distribution for *Tectona grandis* Stands

Model Functions	Maximum Likelihood Estimate				Method of Moments Estimate				
	Loglik.	AIC	BIC	KS	Loglik.	AIC	BIC	KS	Rank
3-P Weibull	-1866.03	3738.07	3751.54	0.0399	-1870.76	3747.52	3760.98	0.0506	1st
Johnson's SB	-1871.34	3750.68	3768.64	0.0459	-1877.97	3763.93	3781.89	0.0459	2nd
GEV	-1919.86	3845.73	3859.19	0.0503	-1919.86	3845.73	3859.19	0.0503	3rd
Gumbel	-1930.45	3864.89	3873.87	0.0553	-1932.09	3868.18	3877.16	0.0636	4th
Logistics	-2004.42	4012.84	4021.81	0.1166	-2008.86	4021.72	4030.65	0.1121	5th
2-P Weibul	-2008.99	4021.98	4030.95	0.1295	-2013.02	4030.04	4039.02	0.1139	6th

Where: Loglik = Log-Likelihood; GEV = Generalized Extreme Value; AIC = Akaike's Information Criterion; BIC = Bayesian Information Criterion; and KS= Kolmogorov-Smirnov statistic

Maximum Likelihood Estimation *Methods of Moments*

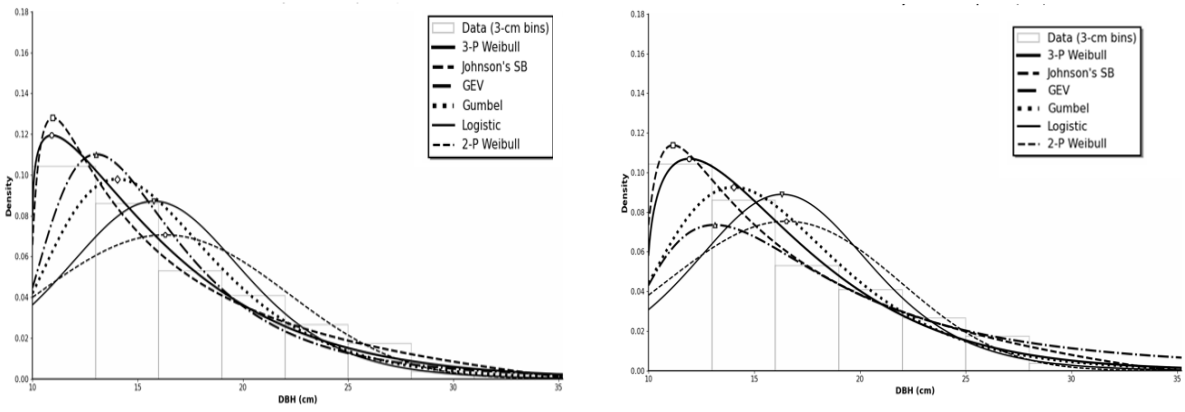


Figure 1: Diameter Distribution Probability Density Function Comparison

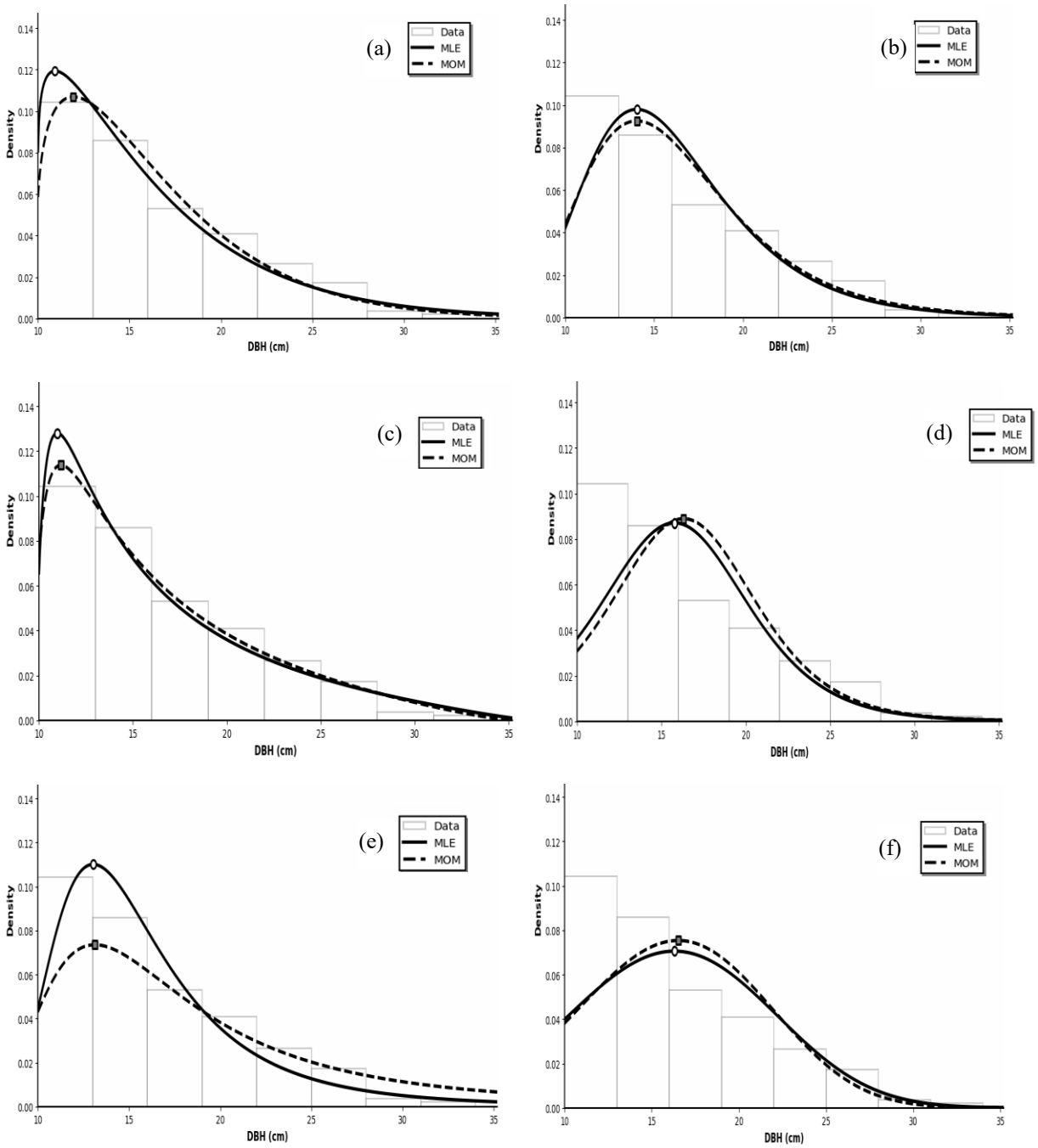


Figure 2(a-f): Probability Density Function Comparison between Methods of Moments and Maximum Likelihood Estimation Approach for 3-P Weibull (a); Gumbel (b); Johnson's SB (c); Logistic (d); Generalized Extreme Value (e) and 2-P Weibull (f)

DISCUSSION

Accurate characterisations of the tree diameter distributions is a fundamental element in growth and yield modelling and are critical for sound decision-making for sustainable management of forest plantations. The diameter characterisation in the study area was examined to support volume estimation and log quality assessment. The variation in the diameters from base to top as showed in the measurements indicates characteristic stem tapering, which is consistently in line with several studies (Bassey and Adekunle, 2022; Popoola *et al.,* 2025). The observed within-stand variability in the diameter structure can be attributed to the heterogeneity in the site quality across the sampled plots (Ashick *et al.,* 2025). The parameter estimates derivative from the Maximum Likelihood Estimate (MLE) and Method of Moments Estimate (MME) approaches showed close agreement for most of the fitted models, indicating that both approaches remain viable for estimating the parameters of diameter distribution models for *Tectona grandis* stands in the study area. The MLE approach consistently yielded marginally smaller standard errors compared to MME, a result that is consistent with the theoretical expectation that MLE estimators attain the Cramer-Rao lower bound asymptotically under standard regularity conditions (Ogana *et al.,* (2016); Gorgoso Varela *et al.,* (2021)). The results for the comparative assessment findings corroborate with the work Gorgoso-Varela *et al.,* (2024) indicating that while both estimation methods perform adequately across the range of stands conditions, MLE demonstrated superior flexibility than the MME under extreme distributional configurations.

Of the six diameter probability distribution models examined, the 3-parameter Weibull equation fitted overall best under both MLE and MME, characterising the diameter structure with a consistent positive skewness. The superiority of the 3-parameter Weibull is in consistent with the work of Zhang *et al.,* (2003); Aigbe, & Omokhua, (2014) and Alo *et al.,* (2017) and confirms its broad applicability across diverse forest types and species (Cao, 2004; Alo *et al.,* 2017; Egonmwam and Ogana, 2020). Although, Johnson's SB model was next to the 3-paramter Weibull distribution model based on performance, there predictive accuracy was close to each other. However, the relative performance superiority of the two models varies across different data sets. Johnson's SB strong performance in this study corroborates with the findings of Ogana *et al.* (2016), Teimouri & Podiaski (2022) who reported that Johnson's SB was among the best-fitting distributions for tropical tree species. Nevertheless, the additional parametric complexity of four parameters Johnson's SB formulation may not always justify its adoption over the parsimonious 3-parameter Weibull, particularly when model simplicity, computational tractability and ease of parameter recovery are prioritised (Bumham and Anderson, 2002).

The inverse J-shaped diameter distribution observed in the pooled stand ages is attributed to the developmental dynamics of the young even-aged stands and also reflects the diameter at breast height measurements set to ≥ 10 cm threshold truncating the left tail. The accumulation of stems in small diameter classes as evidence in this work is similar to the pattern reported by Adedeji, *et al.* (2024) for *Gmelina arborea* in Omo Forest Reserve, Nigeria. The structure of the diameter suggests that there is insufficient early thinning by the management underscoring the need for early systematic thinning operations to promote future structural development of the diameter growth.

The fitted probability density functions (PDFs) curves exhibited biologically plausible trajectories, collectively providing a robust representation of the empirical diameter distribution derived from the detailed stand inventory. These graphical outcomes corroborate the numerical goodness-of-fit statistics, demonstrating capacity both estimation method to reproduce the observed frequency distribution. Although, both the MLE and MME derived curves closely overlapped across the entire Dbh range, the MLE curve more accurately captured the peak frequency and tail behaviour an attribute of particular importance for predicting merchantable volume and stand value across extreme diameter classes.

CONCLUSION AND RECOMMENDATIONS

The 3-parameter Weibull distribution estimated by the Maximum Likelihood Estimation (MLE) provided the best fit for the *Tectona grandis* diameter characterisation in the Area J4 of Omo Forest Reserve, Nigeria. It outperformed all other models examined. Although, both MLE and method of moment estimations were statistically viable, the MLE shows superior flexibility and robustness across the varying stand structures. The stand structure exhibited an inversed J-shaped diameter distribution, with a large number of trees in the small diameter classes. This pattern, though commonly associated with uneven-aged, reflects intense within-stand competition among the stands age under consideration. Thus, requiring an urgent silvicultural intervention.

The 3-parameter Weibull distribution fitted with MLE is therefore recommend for characterising the *Tectona grandis* diameter distribution and projecting the future stand structure in the study area. This measure will support sustainable plantation management and improve both timber quality and yield at the rotation age.

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Authors' Contributions

ODP and SA conceptualized the study. ODP and SOSJ designed the Methodology. ODP performed the data analysis. SA and ODP collected data and wrote the first draft of the

manuscript. ODP provides Supervision. ODP, SA and SOSJ performed literature searches and reviewed the first draft of the manuscript. All authors read and approved the final draft of the manuscript.

Ethical Statement

Not applicable.

AI Declaration Statement

The authors declare that no Artificial Intelligence (AI) tools were used in the generation, analysis, interpretation, or writing of this manuscript.

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